Maximal Acceleration as a Consequence of Heisenberg's Uncertainty Relations.

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Summary. – The existence of a maximal acceleration for physical particles, proposed in 1981 by the author, is shown to be a consequence of Heisenberg's uncertainty relations.

Previous work (¹) on the development of a quantum geometry in relativistic phase space led, quite unexpectedly, to the notion of a *maximal* (*proper*) *acceleration.* This appears because in our theory « particles » are *extended objects,* never to be identified with mathematical points in ordinary space (and it is then immediately intuitive: $a_r = v^2/r \leqslant c^2/r$, which is essentially our formula $A_{\max} \sim c^2/\lambda$, where λ is the « linear dimension » of the particle). The subject has attracted some attention (²), because infinities and collapses would then be forbidden, and in several ways the assumption of a maximal acceleration contains some of the conceptual elements of quantum mechanics.

We propose here to point out that this maximal acceleration appears to be a straightforward consequence of Heisenberg's uncertainty relation

$$\Delta E \cdot \Delta t \geqslant \frac{\hbar}{2}\,, \tag{1}$$

LANDAU was the first, in his theory of fluctuations, to use the consequence of (1)

$$\Delta E \cdot \Delta f(t) \geqslant \frac{\hbar}{2}\left|\frac{\mathrm{d}f}{\mathrm{d}t}\right|\,, \tag{2}$$

(¹) E. R. CAIANIELLO: *Lett. Nuovo Cimento,* **32**, 65 (1981); E. R. CAIANIELLO, S. DE FILIPPO, G. MARMO and G. VILASI: *Lett. Nuovo Cimento,* **34**, 112 (1982).

(²) P. CALDIROLA: *Lett. Nuovo Cimento,* **32**, 264 (1981); H. E. BRANDT: *Lett. Nuovo Cimento,* **38**, 522 (1983); G. SCARPETTA: *Nuovo Cimento,* **41**, 51 (1984).

where $f(t)$ is differentiable. Consider a particle nearly at rest (acceleration can thus be largest [1]) and take $f(t) = v(t)$. Under the assumptions $V \leqslant c$ and

$$\Delta E \leqslant E \,, \qquad \Delta v \leqslant c \tag{3}$$

(2) yields immediately

$$\frac{\hbar}{2}\,|a| \leqslant \Delta E \cdot \Delta v \leqslant mc^2 \cdot c \,,$$

or

$$A_{\max} = 2\,\frac{mc^3}{\hbar} \,. \tag{4}$$

Attitudes vary about the importance to ge biven to small factors such as the 2 in (4). Suppose it should be taken as meaningful. Our previous works gave for $A_{\max}$ (neglecting relativistic factors which cause $A \xrightarrow[\max]{} 0$ when $v \to c$) the equivalent expressions

$$A_{\max} = \frac{\mu^2 c^3}{m\hbar} = \frac{c\hbar}{m\lambda^2} = \frac{\mu c^2}{m\lambda} \tag{5}$$

all deriving from

$$\hbar = \lambda \mu c \qquad \text{(or } h!) \,. \tag{6}$$

Attempts at numerology required the *ad hoc assumption* that $\mu = m$, mass particle, or related somehow to it. λ is then a Compton « diameter », dependent of course on the type of force acting on the particle. The limitation (4) coincides with (5) if we take $\mu = 2m$ (*no more an assumption*); it tells that

$$A_{\max} = \frac{2c^2}{\lambda} = \frac{c^2}{r} \,,$$

which was our intuitive expectation for $v \ll c$. (Limits on higher derivatives, chronons, etc., would follow as easily; we do not pursue here this point, because it might lead, without a deeper scrutiny, to unwarranted conclusions.)

Our work would have been much simpler if we had been able to start the other way around; we confess that we were not then able to « guess » a « maximal acceleration », and found it hard to swallow when first met.

A few words of comment may be in order. Notions such as a « fundamental length » or « time » (chronon) have been and are being proposed as intuitive means to counter the challenge presented by Heisenberg's relations. *No objection* is intended here against such views, especially because they, when pursued with sound physical or mathematical insight, may prove of great interest, whether as facts or tools [3,4]. We wish only to remark that the first revolution came about with Einstein's *maximal velocity*: what proved instrumental to it was not the Kantian *a priori* « space » or « time », but the

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[3] P. CALDIROLA: *Suppl. Nuovo Cimento*, **3**, 297 (1956) and following papers.

[4] L. BIEDENHARN: *Some remarks on using octonions in quantum mechanics*, in *Amalfi Meeting* (May 1983).

« derived » concept of velocity. That was entirely against long-established mental patterns, and it is to be doubted whether, still now, may find it obvious to class « velocity » as an *a priori* concept « more fundamental than space or time.

Such it has of course proved to be, but the decay time of mental patterns lasts generations. With Heisenberg's uncertainty h the situation has not changed; human mind has resisted the second revolution of physics trying to maintain, again, as fundamental the same age old « common sense » concepts as primitive.

The simple thought contained in this note expresses instead the attempt to regard h (as well as c) as « primitive », and all else « derived ». In support of it, we may remark that this attitude brings to a conceptual unification of quantum and information geometry ([5,6]).

The most interesting remark as regards « length » is, in our opinion, that presented by FERRETTI in a recent note ([7]). His argument, as simple as our, proves that the measurement of any length l cannot be more precise than a « quantum noise » lower limit Δl that he computes: it is firm and unobjectionable, since no « fundamental length » is mentioned. Just because of its simplicity, Ferretti's proof adds strength to the question, whether it makes sense to use theories in which lengths smaller than his limit appear (and cause trouble!). Coming to numbers, we remark that, when gravitation is implied, his limit coincides exactly with that obtained from our maximal-acceleration hypothesis, *i.e.* the Planck length ([1]).

It seems to us that the present, naive approach bypasses this type of argumentations. It also appears that in model which should use this notion much has to be re-thought: if for nothing else, because *acceleration* is *gravitation*, and accelerated frames belong already there.

([5]) E. R. CAIANIELLO: *Lett. Nuovo Cimento*, **38**, 539 (1983).
([6]) E. R. CAIANIELLO: *Entropy, information and quantum geometry*, to appear in *Proceeding of Santa Fe International Conference on Nonlinear Phenomena* (July 1984).
([7]) B. FERRETTI: *Lett. Nuovo Cimento*, **40**, 169 (1984).